UDC 539.234:
535.39

BBC

1969
THE QUEEN'S AWARD
TO INDUSTRY

RESEARCH DEPARTMENT REPORT

The theoretical design of multilayer dielectric filters

No. 1969/19

Research Department, Engineering Division
THE BRITISH BROADCASTING CORPORATION

RESEARCH DEPARTMENT

THE THEORETICAL DESIGN OF MULTILAYER DIELECTRIC FILTERS

Research Department Report No. 1969/19
UDC 539.234:
535.39

It uses SI units in accordance with B.S. document PD 5686.

D. Maurice

M.K.E. Smith

Head of Research Department

(PH-35)

Research Department Report No. 1969/19

THE THEORETICAL DESIGN OF MULTILAYER DIELECTRIC FILTERS

(PH-35)

August 1969

Research Department Report No. 1969/19
UDC 539.234:
535.39

THE THEORETICAL DESIGN OF MULTILAYER DIELECTRIC FILTERS

SUMMARY

The principle of operation of multilayer dielectric filters and one method of computing their spectral transmission characteristics is described. The variation of characteristics with number of layers, refractive-index and angle-of-incidence are discussed and it is shown how computer optimization techniques may sometimes be used to obtain required characteristics.

1. INTRODUCTION

The use of multilayer dielectric filters represents the most efficient method so far devised, for splitting incident light into red, green and blue components; as such these filters are used in colour television cameras and telecine equipments where the efficiency of transfer of light to the receptors is of utmost importance in obtaining a good signal-to-noise ratio in the electrical output.

The practical construction of these filters by means of vacuum evaporation is described in an earlier report.[1]

The principles of operation of these filters and the theoretical determination of their transmission characteristics are discussed in this report. The object of the work was to understand the behaviour of such filters and to find a method of designing them to fit the requirements of colour television. This is by no means a complete account of all the possible variations in design of filter, but those designs which have been considered of use in television are discussed.

2. PRINCIPLE OF OPERATION

A multilayer dielectric filter consists essentially of a transparent substrate (e.g. glass) upon which are deposited uniform layers of transparent dielectric materials. Two or more materials of different refractive index may be used. It is found, however, that many different characteristics can be achieved by using only two materials (viz of high- and low-indices deposited alternately) and this report is restricted to consideration of such filters. The thickness of these layers is usually in the range 0·1 to 1 micron. Fig. 1 shows an arrangement of three layers constructed from materials having indices n_A and n_B. An incident ray is reflected, as shown, at the boundaries between media of different refractive index and, depending upon the relative phase and amplitude of the reflected rays, constructive- or destructive-interference occurs. The rays are shown at oblique incidence for clarity.

Let e be the physical thickness of a particular layer. Then its 'optical thickness' is defined to be ne, where n is the refractive index. Let λ_0 be the 'design wavelength' at which the filter is required to

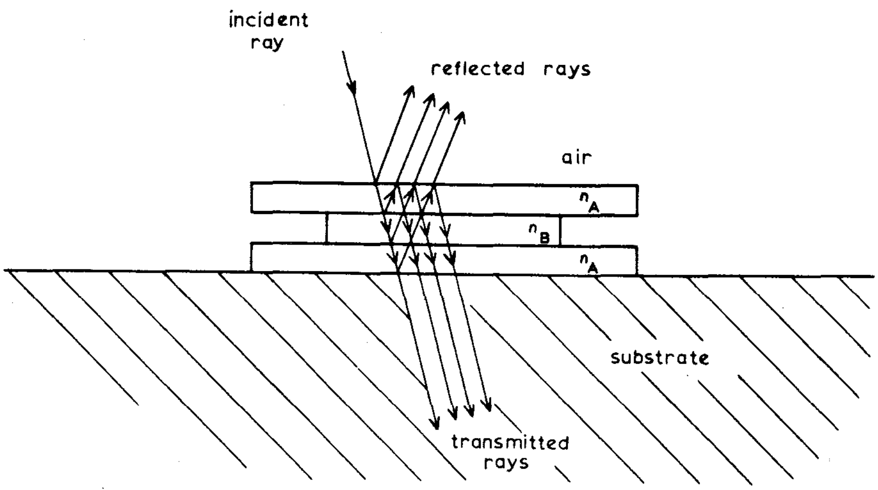

Fig. 1 - Schematic diagram of a three layer filter

have peak reflection. Then if:

$ne = (2k-1)\ \lambda_0/4$ where k is an integer

there is said to be interference of order 'k'. For first-order interference, $k=1$ (the simplest case) and the optical thickness is $\lambda_0/4$. Fig. 2 shows a typical theoretical spectral transmission characteristic for which the design wavelength is 700 nm; it is constructed from zinc sulphide and magnesium fluoride which are high- and low-refractive index materials respectively. The solid curve is the characteristic obtained when the dispersion (or variation of refractive index with wavelength of incident light) of each material is taken into account. The effect of ignoring dispersion is shown, for comparison, by the broken-line curve.

At the design-wavelength, the transmission is at a minimum and therefore the reflection at a maximum; as the wavelength is shortened the transmission rises, gradually at first and then rapidly to a maximum. Over the transmitting region secondary minima occur; this oscillatory nature of the characteristic can be undesirable. Methods of improving the characteristic are detailed in Section 8.

Since the two curves of Fig. 2 are very similar in shape and dispersion data are not known for all evaporated materials, dispersion has been neglected in the remainder of this report (except for Figs. 15 and 16). If the transmission characteristic (at normal incidence) is plotted in terms of wave number (i.e. reciprocal of wavelength) then the curve is almost symmetrical about the design wavenumber and is precisely symmetrical if dispersion is ignored (see Fig. 3). This symmetry holds even if the filter contains layers of integrally related thicknesses, such that different orders of interference are taking place in the various layers.

3. METHOD OF COMPUTING SPECTRAL CHARACTERISTICS

One possible method of computing performance depends upon the analogy between a multilayer filter and a series of transmission lines having different lengths and different characteristic impedances. Admittance is replaced by refractive index and optical thickness is substituted for length of line. The

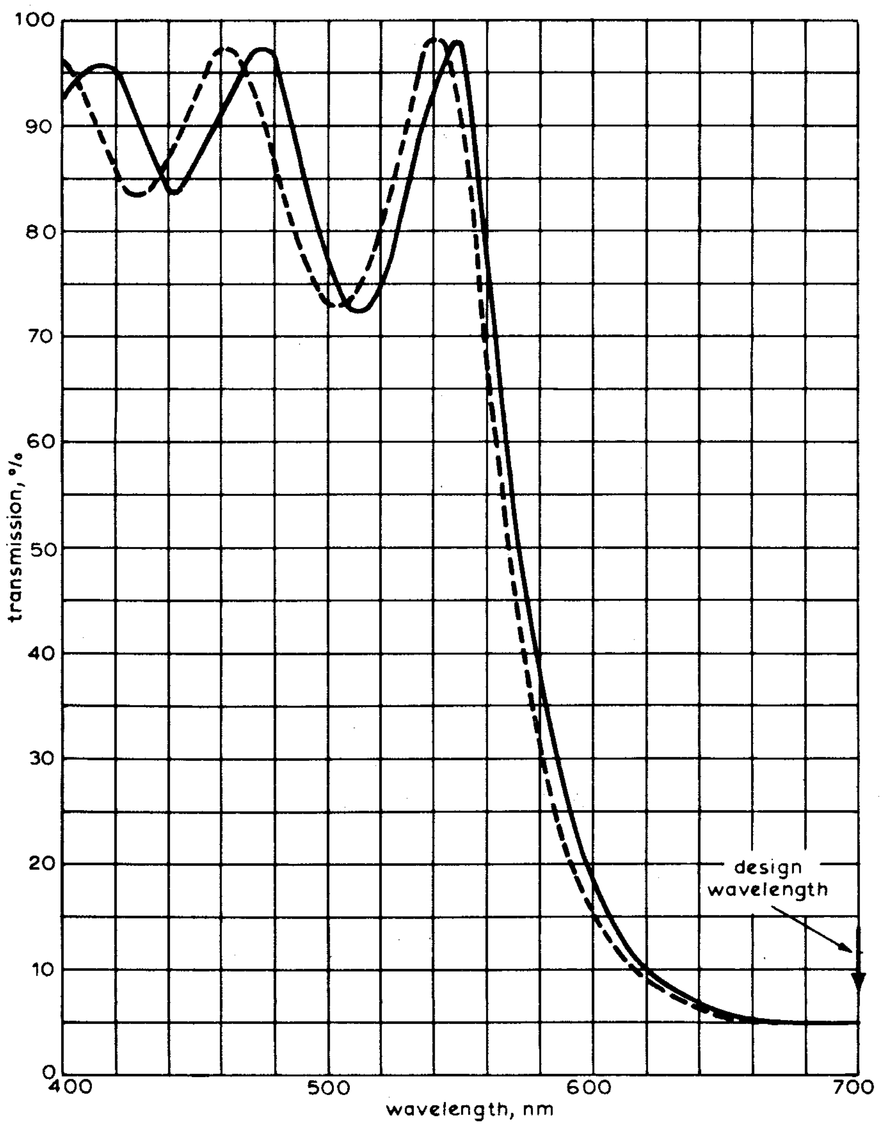

Fig. 2 - Theoretical spectral transmission characteristic of a typical first-order-interference filter

Seven layers $n_H = 2{\cdot}35$ $n_L = 1{\cdot}38$ $n_G = 1{\cdot}52$ $\lambda_0 = 700$ nm

----- No dispersion
——— With dispersion

method used here involves the transmission matrix for an electro-magnetic wave travelling through a medium; a succession of layers is represented as a succession of media and by matrix multiplication a transmission matrix for the whole stack is obtained. By inserting the appropriate boundary conditions at the substrate, the complex (amplitude) reflection coefficient is derived and hence the intensity (power) coefficient. The formal mathematics can be stated as follows:

For one layer at normal incidence

$$E = E_0 \cos\beta + H_0 \,(\mathrm{j}\sin\beta)/n$$
$$H = E_0 \,\mathrm{j}\, n \sin\beta + H_0 \cos\beta$$

where E and H are the electric and magnetic components of the incident wave near to the boundary with the thin film.

n is the refractive index of the layer,
e is the physical thickness of the layer,
λ is the wavelength of the incident light (in vacuo),
$\beta = 2\pi ne/\lambda$ = change of phase of wave in travelling through the thin film (at normal incidence) and

E_0 and H_0 are the electric and magnetic components of the wave at a point adjacent to the boundary and in the substrate.

Thus $H_0 = n_0 E_0$ where n_0 = refractive index of the substrate.

These equations for E and H can conveniently be combined into the single matrix equation:

$$\begin{bmatrix} E \\ H \end{bmatrix} = \begin{bmatrix} M_1 \end{bmatrix} \begin{bmatrix} E_0 \\ H_0 \end{bmatrix}$$

where

$$\begin{bmatrix} M_1 \end{bmatrix} = \begin{bmatrix} \cos\beta & (\mathrm{j}\sin\beta)/n \\ \mathrm{j}\, n \sin\beta & \cos\beta \end{bmatrix}$$

If there are x layers, the matrices for each layer are multiplied so that:

$$\begin{bmatrix} E \\ H \end{bmatrix} = \begin{bmatrix} M_x \end{bmatrix} \begin{bmatrix} M_{x-1} \end{bmatrix} \cdots\cdots \begin{bmatrix} M_1 \end{bmatrix} \begin{bmatrix} E_0 \\ H_0 \end{bmatrix}$$

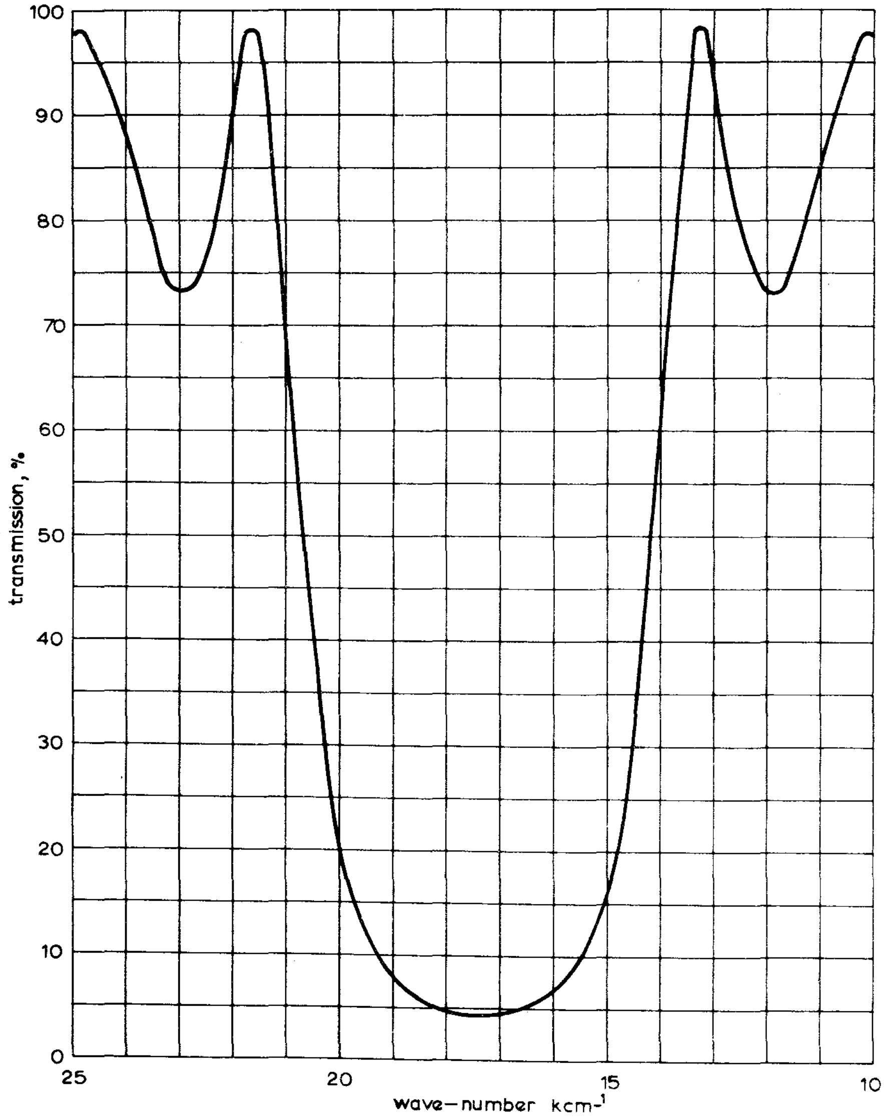

Fig. 3 - Showing symmetrical nature of characteristic when plotted in terms of wave-number

Seven layers $\lambda_0 = 575$ nm (wavenumber = 17391 cm^{-1})
$n_H = 2{\cdot}35$ $n_L = 1{\cdot}38$ $n_G = 1{\cdot}52$

The admittance Y of the whole stack is given by:

$$Y = \frac{H}{E}$$

and by substituting $H_O = n_O E_O$, as explained above, a complex expression for Y is obtained, containing the refractive indices and optical thicknesses of the layers and the refractive index of the substrate.

The complex amplitude reflection coefficient r, is given by:

$$r = \frac{m - Y}{m + Y}$$

where m is the refractive index of the medium in which the filter is immersed. Thus the intensity reflection coefficient, R^2 is given by

$$R^2 = rr^*$$

where the asterisk denotes the complex conjugate.

At oblique incidence the spectral response of a filter depends upon the plane of polarization of the incident light.

For the component perpendicular to the plane of incidence (s component), the characteristic of a filter may be determined by replacing n_x by $n_x \cos\theta_x$ and for the component parallel to the plane of incidence by $n_x/\cos\theta_x$,

where n_x = refractive index of the xth layer
and θ_x = angle between a ray in this layer and the normal.

This substitution takes account of the reflection coefficient at a boundary for a stated angle of incidence.

The phase change (allowing for the decrease in effective thickness) is obtained by putting

$$\beta = (2\pi/\lambda)\ n_x e \cos\theta_x$$

For an accurate calculation of the spectral characteristics of any filter, the dispersion of the materials in each layer must be taken into account. The refractive index n at any wavelength λ may be found by using the Cauchy equation

$$n = A + B/\lambda^2 + C/\lambda^4$$

where A, B and C are constants for a given material and may be determined, provided the refractive index of the material is known at three wavelengths.

A programme has been written in Autocode for use on the Elliott 803 computer, which enables the spectral characteristic of any design of filter to be readily evaluated. The computational method is as outlined above and takes account of oblique incidence and dispersion when required.

Although, in practice, the surface of a substrate which is not perfectly anti-reflection coated will contribute some reflection, this has been ignored in the programme; the maximum effect of this for a glass substrate is to show a theoretical transmission which is about 4% higher than would be obtained for a practical filter.

4. DESIGN PARAMETERS

It can be shown that the reflection coefficient R^2 at the design-wavelength, of a stack of p layers (where p is an odd integer) is given by:

$$R^2 = \left[\frac{n_L^{p-1} n_G - n_H^{p+1}}{n_L^{p-1} n_G + n_H^{p+1}}\right]^2 \qquad (1)$$

where n_L = refractive index of low index layer,
n_H = refractive index of high index layer
and n_G = refractive index of substrate

For a filter constructed from given materials, the peak reflection coefficient increases with the number of layers, provided that this number is increased by two or multiples thereof; the addition of one quarter-wave layer of low-refractive-index to a stack consisting of an odd number of layers will only result in a decrease in peak reflection.

The peak reflection obtained for a given number of layers will depend upon the refractive-index of the materials used to construct the filter; the relationship is given in equation (1).

This report considers only the effects of variations of the index of the high-refractive-index layers. A satisfactory material has been found for the low-refractive-index layer and this has been used in all filters so far constructed.

Table 1 shows the calculated values of index of the high-refractive-index layers which are required to produce filters having 95% peak reflection, for numbers of layers from 5 to 19; the values of refractive-index of the glass and low-refractive-index layers are given.

TABLE 1

Number of Layers	Refractive index of high-index layers required for exactly 95% reflection $n_L = 1{\cdot}38 \quad n_G = 1{\cdot}52$
5	2·75
7	2·31
9	2·08
11	1·95
13	1·85
15	1·79
17	1·74
19	1·70

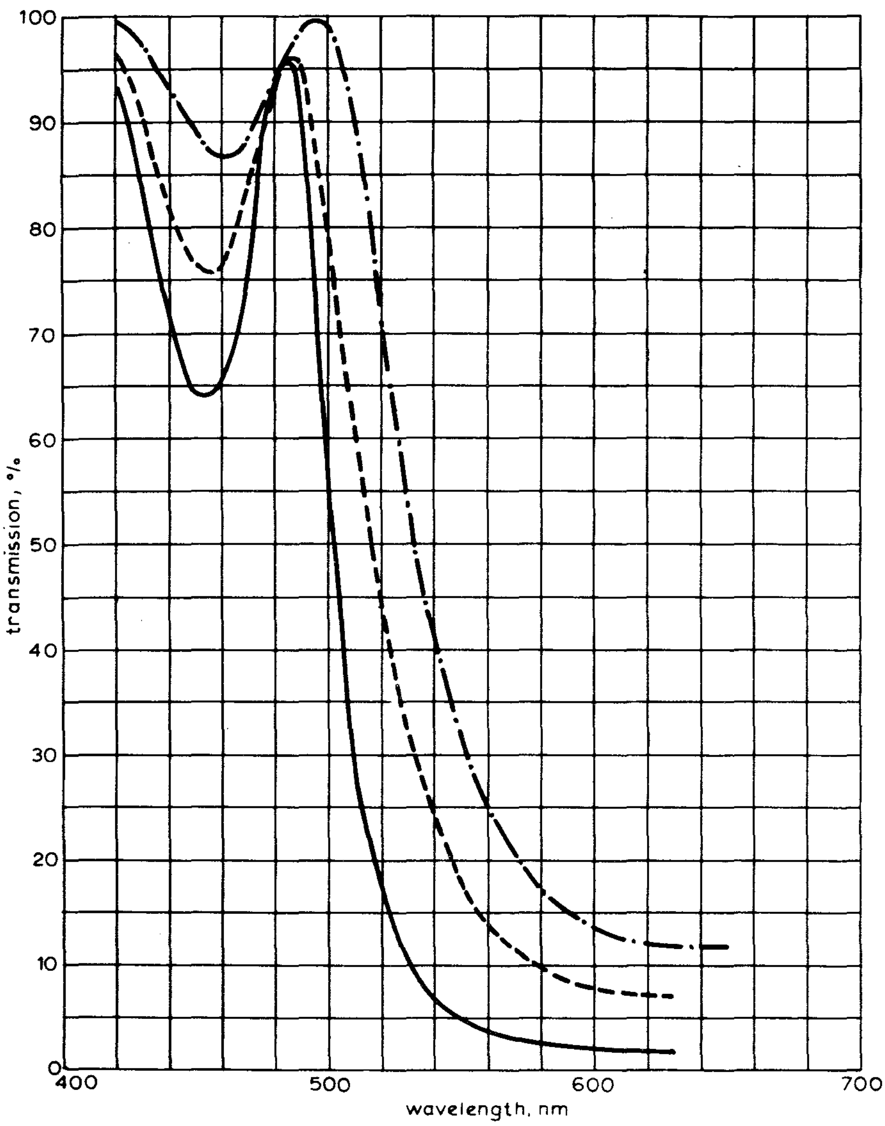

Fig. 4 - Spectral transmission characteristics of 7 layer filter to s and p polarizations at 45 degrees angle of incidence

$n_H = 2{\cdot}35$ $n_L = 1{\cdot}38$ $n_G = 1{\cdot}52$

——— *s* polarization
—·—·— *p* polarization
- - - - - Mean $= \frac{s+p}{2}$

In practice, of course, one cannot usually find a material which has exactly the required refractive index; the table is only intended as a guide to the number of layers necessary in a filter, for any given material.

Filters used in television require a high reflection coefficient, but not necessarily the highest possible, as do those used as laser mirrors. A figure of 95% reflection is a reasonable compromise between reflection and number of layers; the difficulty of construction and the time required for it increase with the number of layers. Furthermore, filters used as dichroic mirrors are often working at 45° incidence; at angles such as this, high reflection multilayers may exhibit undesirable characteristics; these effects are discussed in Section 5.

Another significant feature of multilayer dielectric filters, is the rate of the transition between the transmitting and reflecting mode, with change of wavelength. This may be conveniently measured as the slope at a point on the spectral characteristic halfway between peak transmission and peak reflection; for high reflection filters this may be taken as the 50% transmission point.

5. FILTERS AT OBLIQUE INCIDENCE

As mentioned in Section 3, the spectral transmission characteristic of these filters at oblique incidence depends upon the plane of polarization of the incident illumination; hence the characteristic for ordinary non-polarized light (and for circularly-polarized light) is the mean of the characteristics for light which is polarized parallel to and perpendicular to the plane of incidence (i.e. mean of p and s components respectively.) The effects are shown in Figs. 4 and 5 for filters at 45° incidence. Fig. 4 shows the spectral transmission characteristics of a seven layer filter for s and p polarizations; the mean curve is shown and it is seen that the shape of these curves is similar to those obtained at normal incidence (Fig. 2), with a shift of peak reflectance to a shorter wavelength.

Fig. 5 shows the corresponding characteristics for a filter having 11 layers, which would give a peak reflection at normal incidence of over 99%. It can be seen that in the region of spectrum where the filter is transmitting, the secondary minimum and first maximum for each polarization are further 'out of phase'

than in the case of the filter in Fig. 4. In consequence the mean curve has a low value of peak transmission and the transition from reflection to transmission is no longer smooth, but has a point of inflection.

6. FIRST ORDER FILTERS

6.1. Variation of High-refractive-index Layers

The spectral transmission characteristics have been computed for five first order interference filters having different values of index for the high-refractive-index layers. These are shown in Figs. 6 and 7, for normal and 45° incidence respectively. All filters have the same design-wavelength of 700 nm, and at 45° the curve shown is the mean of *s* and *p* polarizations. To avoid confusion, some of the curves have been drawn only as far as the secondary minimum. The refractive indices chosen for the high-refractive-index layers are those which can be obtained using known materials, with the possible exception of the highest value (2·5) for which a material is known, but which it has only been possible to deposit as one layer with our present facilities. The number of layers has been chosen in each case to give a peak reflectance as near as possible to 95%. The dispersion of each material has been neglected.

6.2. Effect on Slope of Transition

It can be seen from Fig. 6 that a large number of layers in a filter having a relatively low value of index for the high-refractive-index layers produces a characteristic with a higher rate of transition from the reflecting to the transmitting mode than a small number of layers with a high value of index.

The slope at the 50% transmission point has been measured on each curve; the results are shown graphically in Fig. 8. The slope increases rapidly as the index decreases, so that a very steep slope should be obtained from a filter constructed from materials having close values of refractive-index, although this would involve the deposition of many layers. Furthermore the monitoring of such a filter would pose difficult problems.[1]

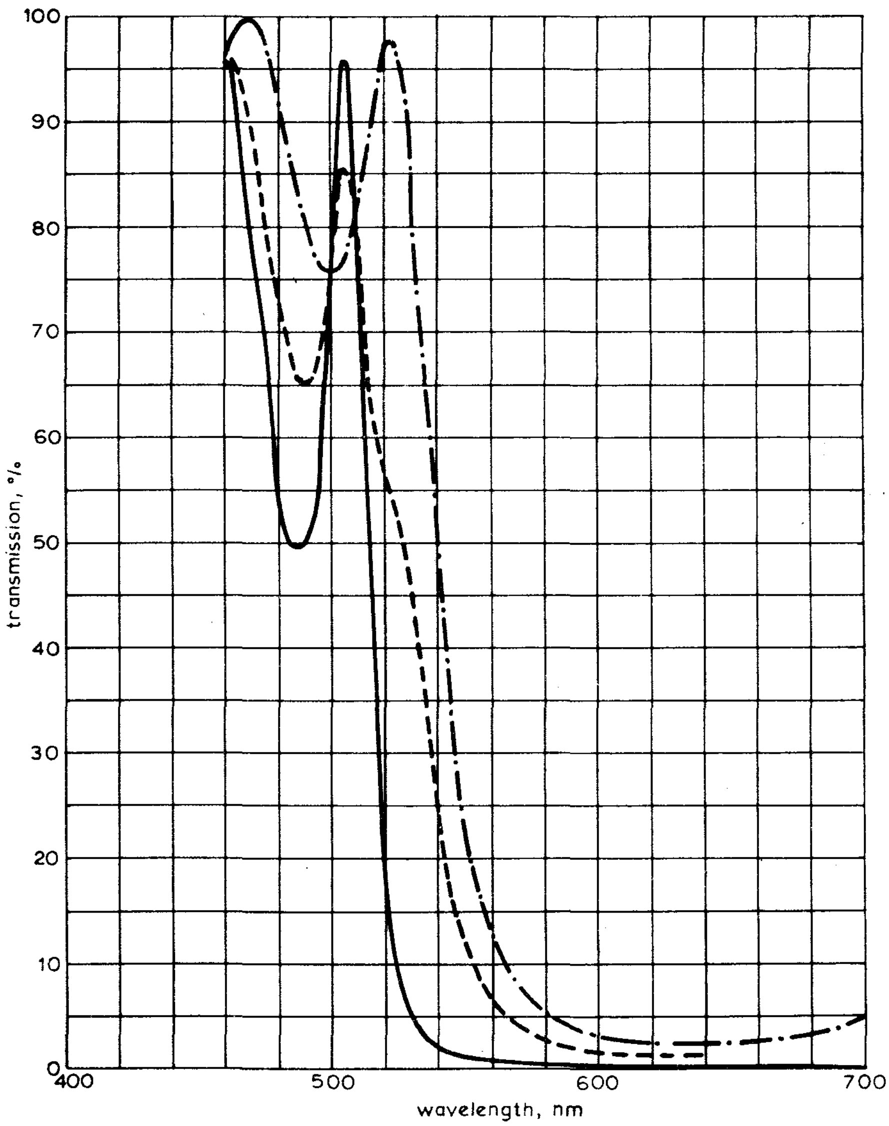

Fig. 5 - Spectral transmission characteristic of 11 layer filter to s and p polarizations at 45 degrees incidence

$n_H = 2{\cdot}35$ $n_L = 1{\cdot}38$

——— *s* polarization
—·—·— *p* polarization
- - - - - Mean $= \frac{s+p}{2}$

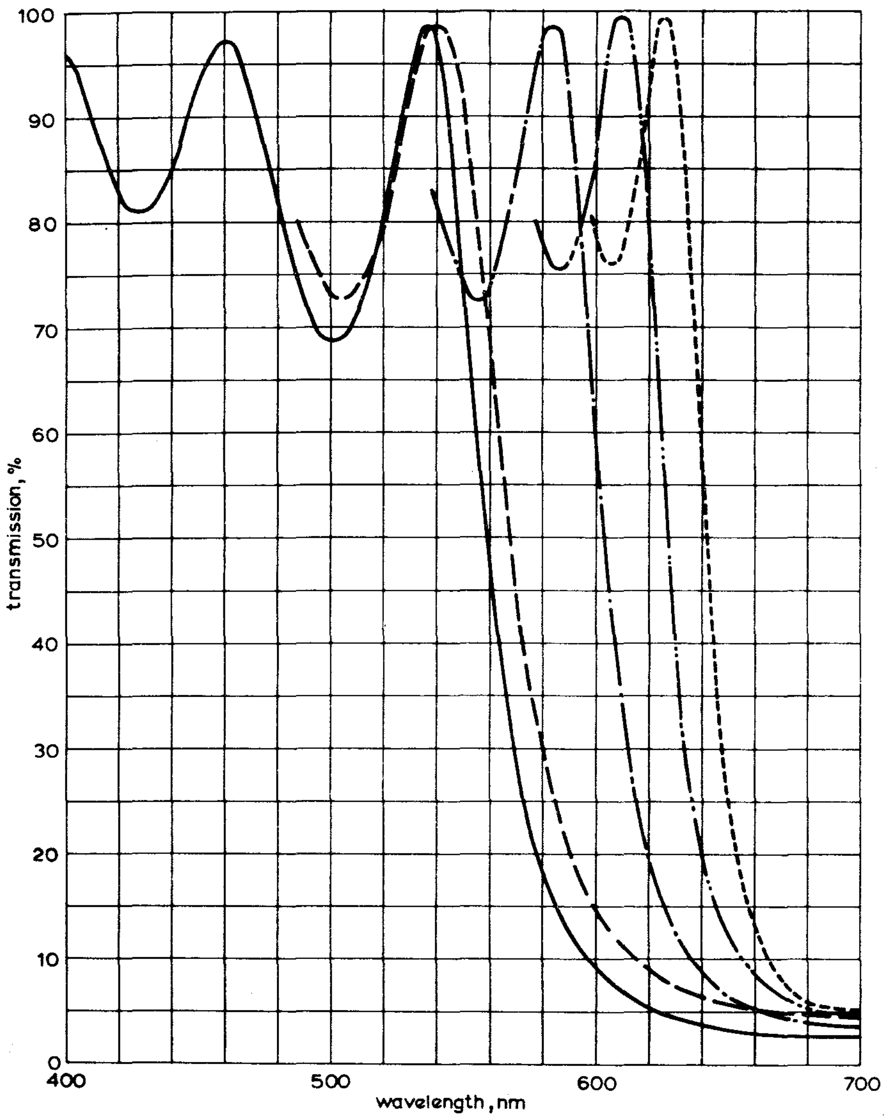

Fig. 6 - Spectral transmission characteristic of first-order-interference filters with various indices for the high-refractive-index layers

$n_L = 1{\cdot}38$ $n_G = 1{\cdot}52$ $\lambda_0 = 700$ nm

No dispersion, Normal incidence

$n_H = 2{\cdot}5$, 7 layers
$n_H = 2{\cdot}35$, 7 layers
$n_H = 2{\cdot}0$, 11 layers
$n_H = 1{\cdot}8$, 15 layers
$n_H = 1{\cdot}7$, 19 layers

6.3. Reflection Bandwidth

As the slope of transition between transmission and reflection decreases, the bandwidth of the reflecting region of the characteristic increases. It is convenient to measure the half-bandwidth* from the design-wavelength to the 50% transmission point; the curve of Fig. 9 shows the way in which this varies with the index of the high-refractive-index layers.

6.4. Shift of Peak Reflectance at Oblique Incidence

Referring to Fig. 7 it can be seen that the peak reflectance of each filter is shifted to a shorter wavelength; the magnitude of this shift depends upon the value of refractive-index used in the high-index layer, being least for the highest value of index. In order to determine the exact position of the peak it was found necessary to evaluate the characteristics of the filters at intervals of 1 nm in the region of the peak. A graph of the variation of the shift of wavelength of the peak with refractive-index is shown in Fig. 10.

It must be emphasized that the curves given in Figs. 8, 9 and 10 apply only to first-order filters with a design-wavelength of 700 nm and a reflectance of about 95% at this wavelength.

7. FILTERS WITH HIGHER ORDER INTERFERENCE LAYERS

All filters so far described in this report are of the first-order interference type where the optical thickness of all layers is one-quarter the design-wavelength. If the optical thickness of the high-refractive-index layers is increased to three-quarters of the design-wavelength, then second-order interference takes place. Similarly third-order interference filters have high-refractive-index layers of optical thickness one-and-a-quarter times the design-wavelength. Orders higher than third are of limited interest because in practice it is difficult to deposit relatively thick

* The full bandwidth (to the 50% point) is not twice the figure quoted in Fig. 8 because the non-dispersive version is symmetrical in wave-number rather than wavelength.

layers of dielectric material in multilayer constructions owing to mechanical instability caused by stresses set up in the layers.

The general effect of increasing the order of interference (in the high index layer) is to increase the slope of the spectral characteristic and to decrease the reflection bandwidth. With filters of this type and a design-wavelength at the long-wavelength end of the visible spectrum, second reflection peaks occur in the visible region. These are caused by higher order interference occurring at a shorter wavelength than the design-wavelength. Thus if second-order interference is occurring in the high-index layers at wavelength λ_O, then third-order interference would be expected to occur at wavelength λ where

$$\frac{3\lambda_O}{4} = \frac{5\lambda}{4}$$

thus if $\lambda_O = 700$ nm then, $\lambda = 420$ nm.

However the effect of the low-refractive-index layer (which is of thickness such as to cause first-order interference) is to shift this peak to a longer wavelength and because the layers are not all exact multiple quarters of this wavelength, the peak value of reflection factor obtained is lower than at the design-wavelength.

Figs. 11 and 12 show the spectral transmission characteristic of second- and third-order filters with a design-wavelength of 700 nm; these have seven layers, the high-index layer having a refractive-index of 2·35.

The second-order filter shown has a second reflection peak at a shorter wavelength of about 458nm. This type of filter is only suitable for use where this second peak will not have a detrimental effect. The slope of the filter at the 50% point is 3·3% per nm which is the same as in a first-order filter having fifteen layers with a high-index layer of refractive index 1·8 (Fig. 8).

The third-order filter in Fig. 12 has two extra reflection peaks and this also has limited use; however the slope is very high at 5% per nm, which is very useful as a sharp cut-off filter if the narrow transmission bandwidth can be tolerated.

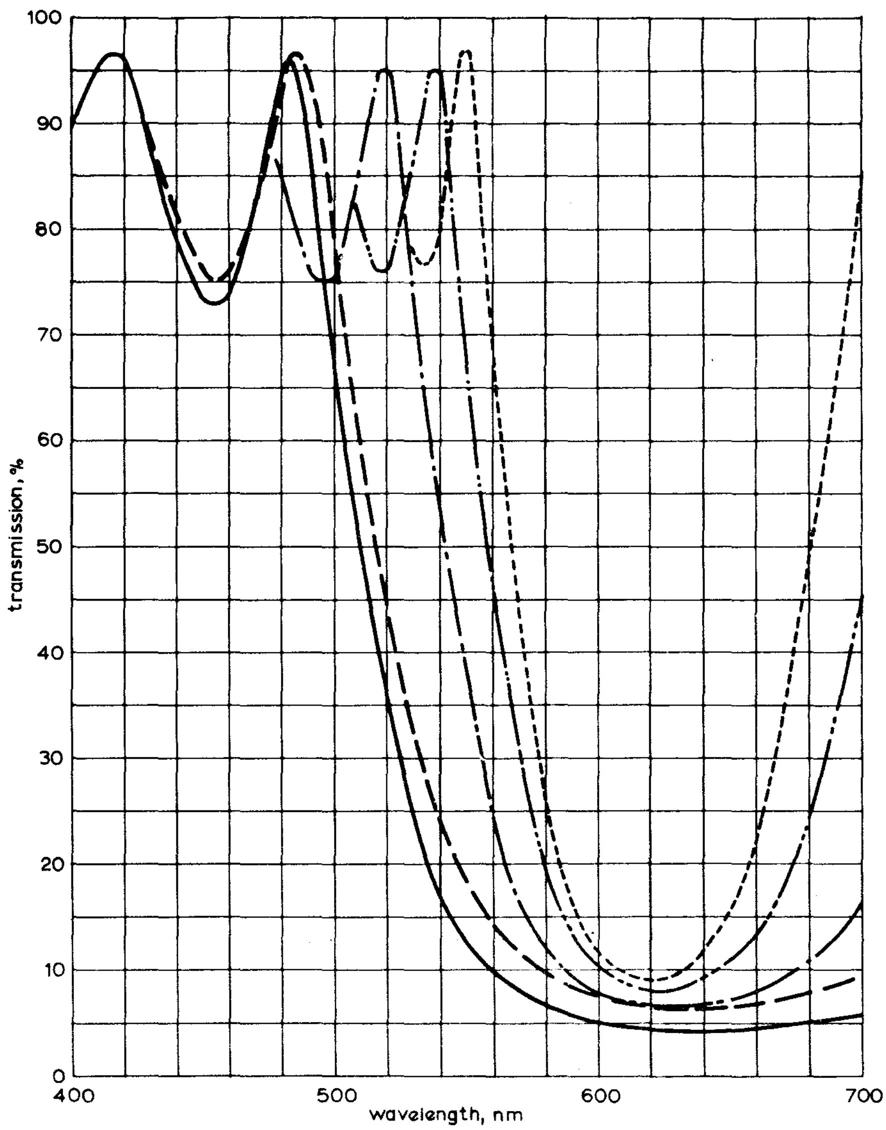

Fig. 7 - Spectral transmission characteristic of first-order-interference filter with various indices for the high-refractive-index layers

No dispersion 45 degrees incidence
Mean of s and p polarizations
$n_L = 1·38$ $n_G = 1·52$

- $n_H = 2·5$, 7 layers (solid line)
- $n_H = 2·35$, 7 layers (dashed line)
- $n_H = 2·0$, 11 layers (dash-dot line)
- $n_H = 1·8$, 15 layers (dash-double-dot line)
- $n_H = 1·7$, 19 layers (short dashed line)

If the design-wavelength of a second-order filter is put at the short wavelength end of the visible spectrum, the transmission band extends through the visible without the occurrence of a second peak; a useful blue reflecting filter can be made using this design.

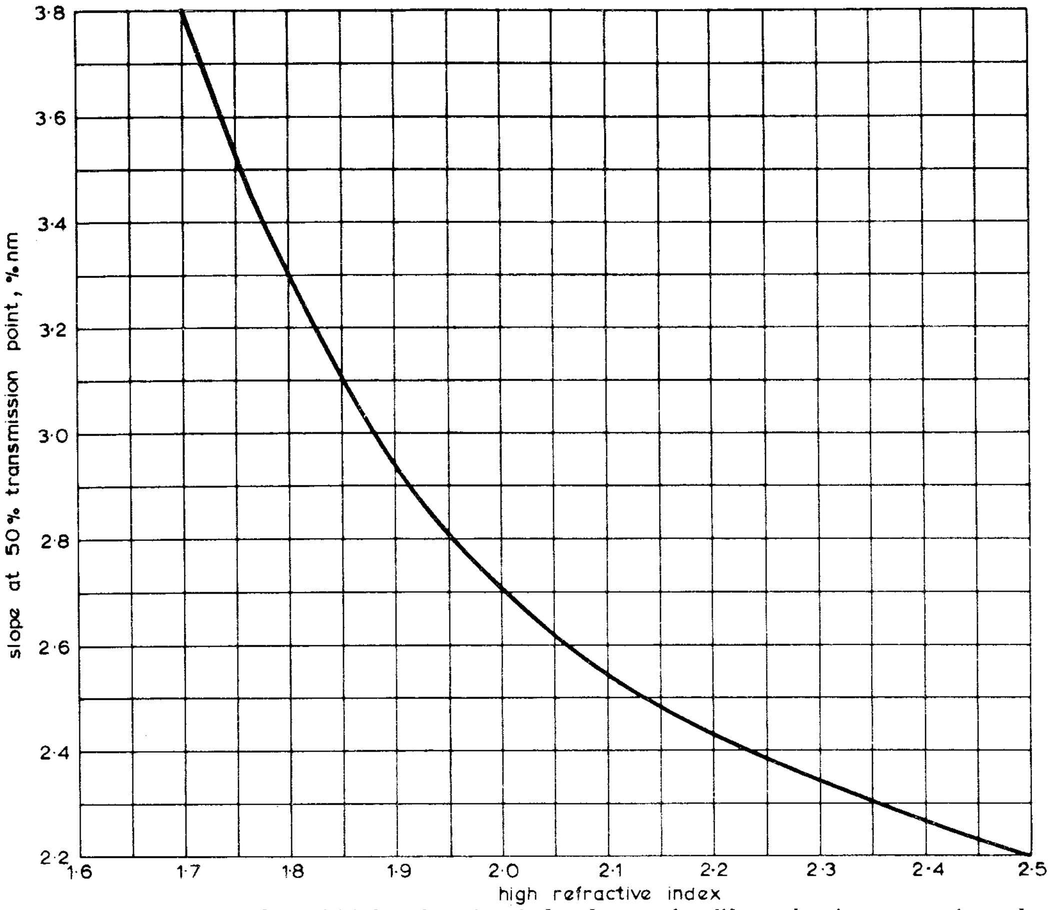

Fig. 8 - Variation of slope with index of high-refractive-index layers for filters having approximately 95% reflection at the design-wavelength

$n_L = 1{\cdot}38 \qquad n_G = 1{\cdot}52$

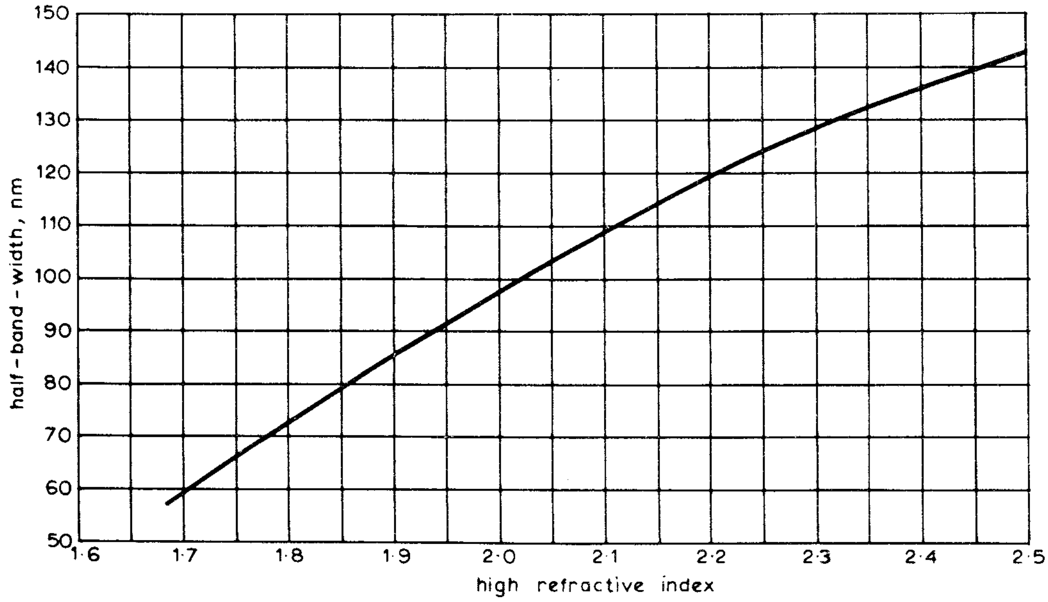

Fig. 9 - Variation of half-bandwidth with index of high-refractive-index layers for filters having approximately 95% reflection at the design-wavelength

$n_L = 1{\cdot}38 \qquad n_G = 1{\cdot}52$

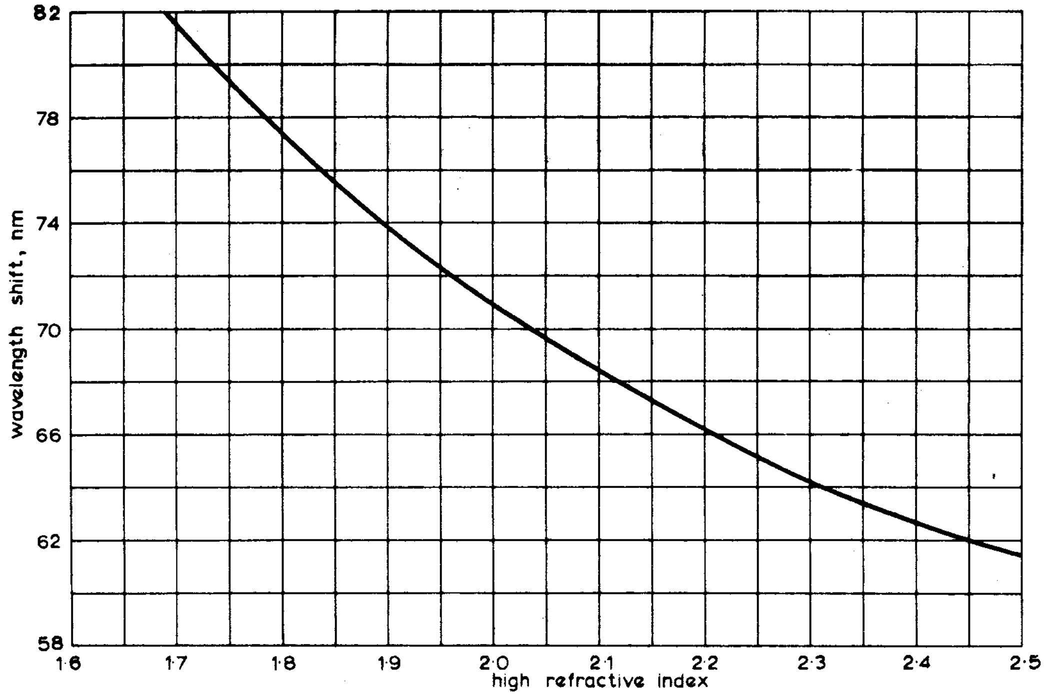

Fig. 10 - Variation of shift of peak-reflectance-wavelength at 45 degrees, with index of high-refractive-index layers for filters having approximately 95% reflectance at the design-wavelength

$n_L = 1{\cdot}38 \qquad n_G = 1{\cdot}52$

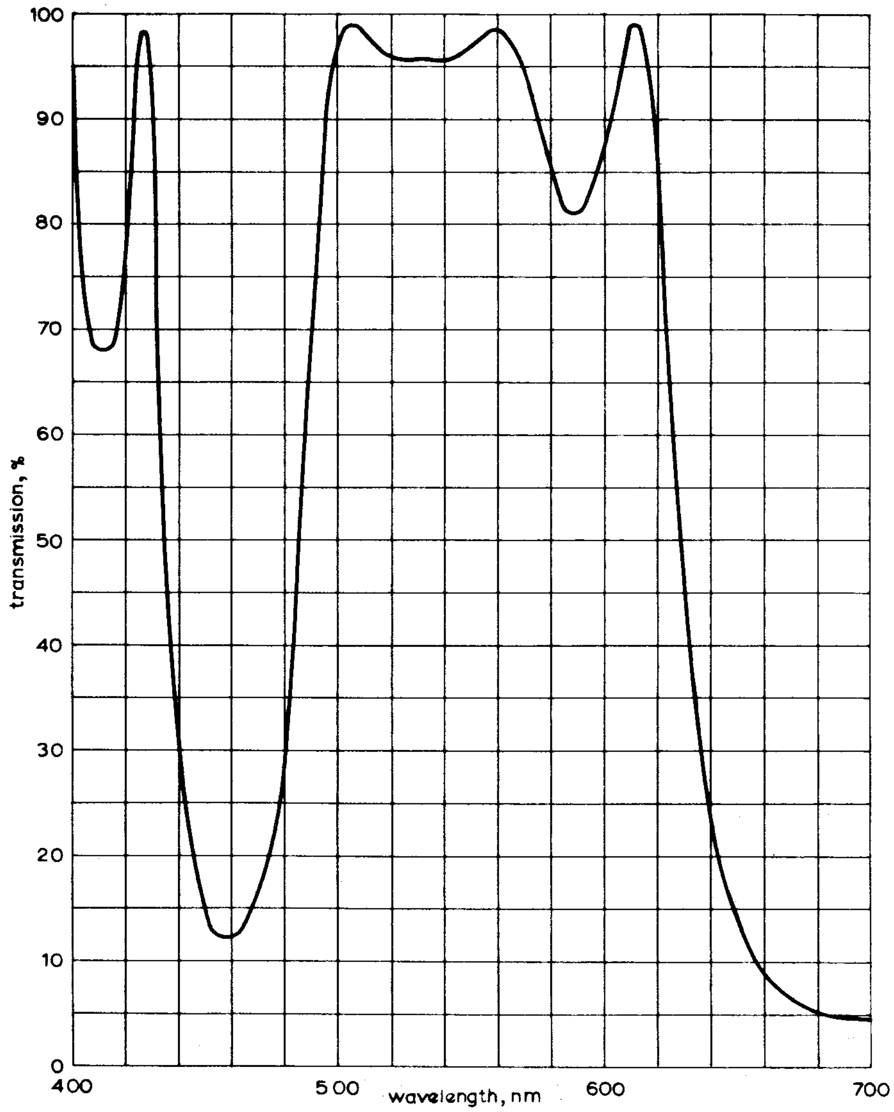

Fig. 11 - Spectral transmission characteristic of a typical second-order-interference filter

Seven layers

$n_H = 2{\cdot}35 \quad n_L = 1{\cdot}38 \quad n_G = 1{\cdot}52$

$\lambda_0 = 700$ nm

8. SMOOTHING OF TRANSMISSION CHARACTERISTICS

As has already been said, the secondary minima and in particular the first minimum, which occur in the transmitting region of the characteristics are undesirable. These can be attenuated in all filters by the addition of a half-wavelength optical-thickness of the low-refractive-index material to the filter.[2] The result of this on a first-order filter is shown in Fig. 13 which shows that a considerable smoothing of the transmission characteristic has been effected; a slight reduction in the slope of the transition is an additional consequence. Higher-order filters are similarly affected and it has been found to work best for second-order and about equally well for first- and third-order filters.

A first-order filter, having a design wavelength of 450 nm and constructed so that its first and last layers are replaced by layers having an optical thickness of one-eighth of the design-wavelength, has a characteristic as shown in Fig. 14. This exhibits a very smooth characteristic, but this time with a significant loss of slope. The characteristic of a filter with the same design-wavelength, constructed with all quarter-wavelength layers is shown for comparison. Another effect of the eighth-wavelength layers is to shift the peak reflection to a shorter wavelength. The characteristic given is for a filter having a short design-wavelength; this is because this filter, unlike other designs, is not symmetrical* about the design-wavelength and although the transmission band on the long-wavelength side of the design-wavelength is smoothed, the amplitude of the secondary minimum on the short-wavelength side is increased. The corresponding solution, if there is one, for filters with a long design-wavelength has not been found.

* Strictly in terms of wave-number for non-dispersive layers.

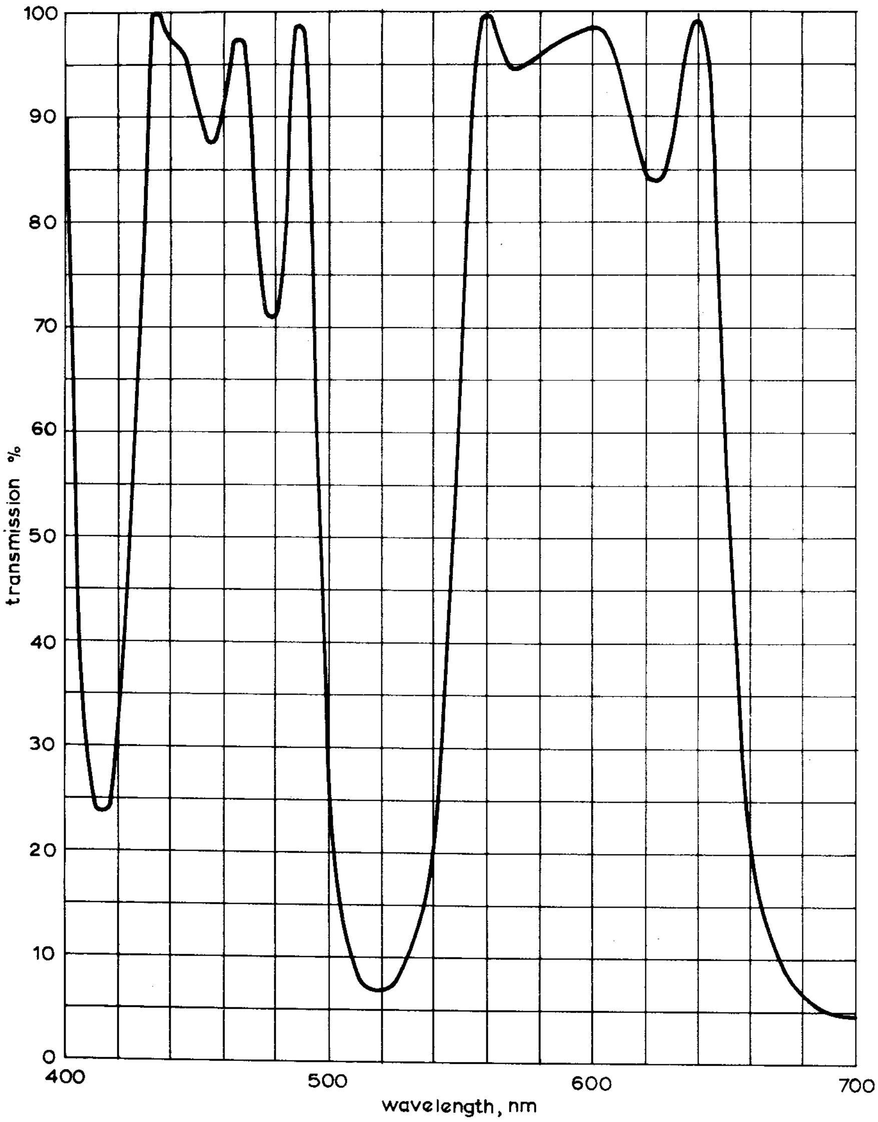

Fig. 12 - Spectral transmission characteristic of a typical third-order-interference filter

Seven layers
$n_H = 2{\cdot}35$ $n_L = 1{\cdot}38$ $n_G = 1{\cdot}52$
$\lambda_0 = 700$ nm

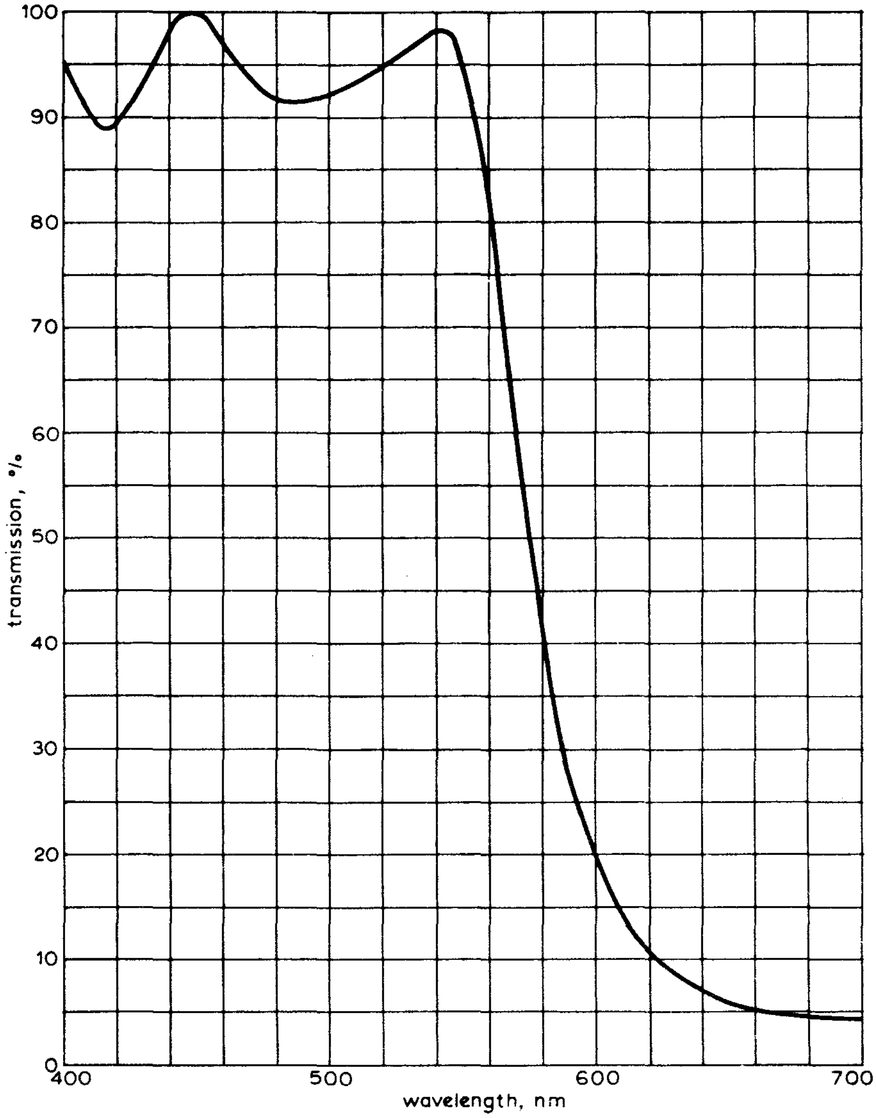

Fig. 13 - Spectral transmission characteristic of a first-order-interference filter with additional $\lambda_0/2$ layer of low-refractive-index

Eight layers

$n_H = 2{\cdot}35$

$n_L = 1{\cdot}38$

$\lambda_0 = 700$ nm

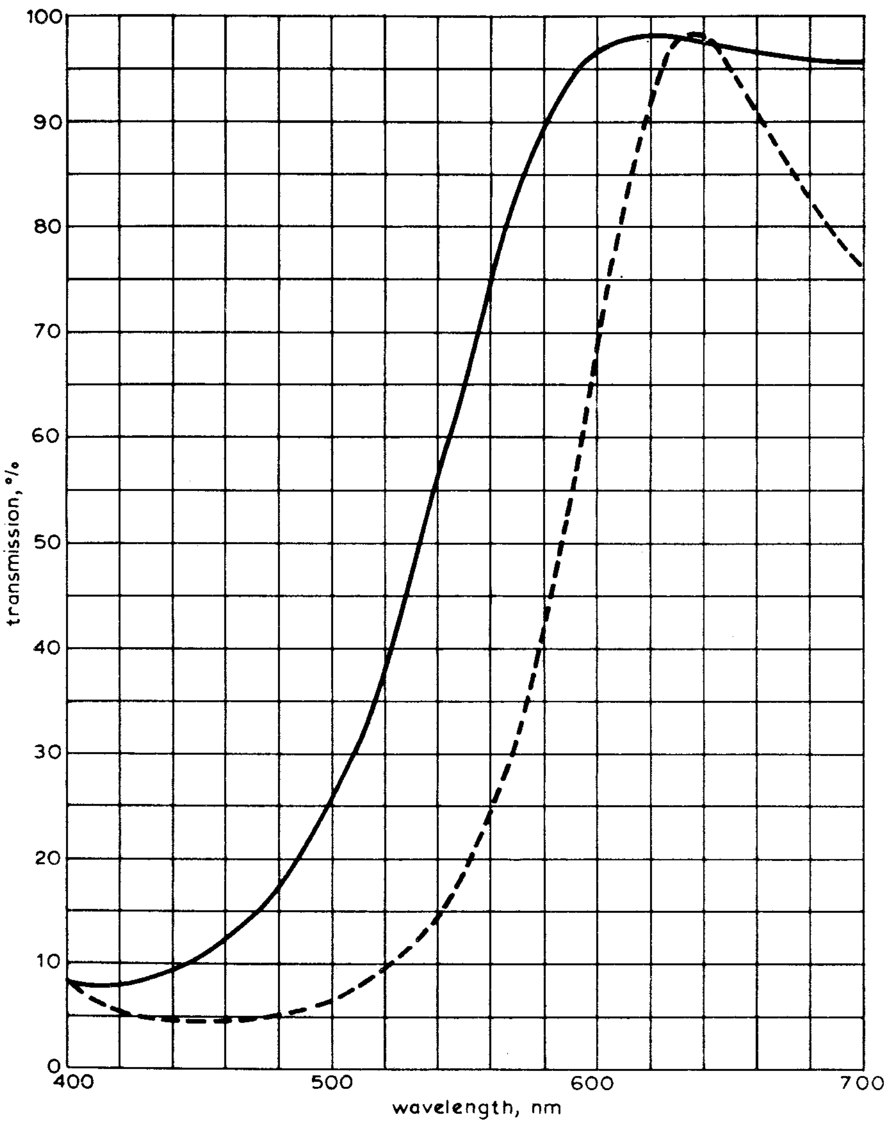

Fig. 14 - Spectral transmission characteristic of a first order filter with first and last layers equal to $\lambda_0/8$

— — — — — Normal first order filter for comparison

Seven layers

$n_H = 2 \cdot 35 \quad n_L = 1 \cdot 38 \quad \lambda_0 = 450$ nm

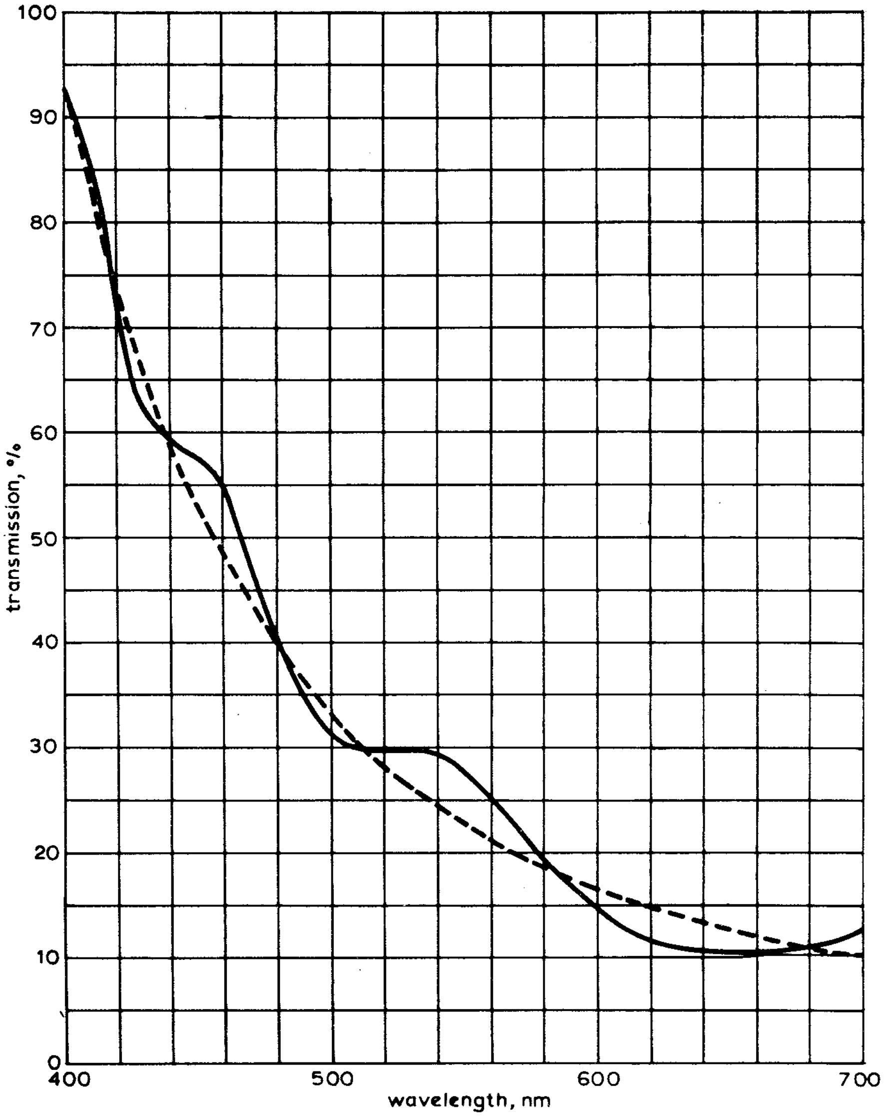

Fig. 15 - Colour temperature raising filter

--- Required characteristic
——— Optimised seven layer filter

With dispersion
n_H = ZnS n_L = MgF_2

9. OTHER DESIGNS OF FILTER

Filters may be constructed in which the optical thicknesses of the layers do not bear a simple relationship to each other or to a single design-wavelength. The spectral characteristic of such a filter may be determined using the computer programme described in Section 3. Additionally by using the computer programme as a sub-routine in the Elliott Autocode Library Program for System Optimization, it has been possible to determine the thickness of layers required to give characteristics other than those described in Sections 6 and 7. This technique does not always work well because (a) a satisfactory solution may not exist and (b) with a large number of parameters to vary, the optimization programme may reach a false minimum.

This technique was successfully used for the production of a colour-temperature-raising filter for quartz-iodine lamps. A gelatine or dyed glass filter was found unsuitable because of the great amount of heat radiated by the lamp, much of which is absorbed in the filter; with a multilayer filter the colour temperature raising was achieved by reflection and not by absorption. In Fig. 15 the characteristic of the multilayer filter is compared with the required response; the practical filter was constructed (with some difficulty owing to the absence of a common design-wavelength) and proved successful in use.

The technique has also been used in an attempt to smooth the characteristic of filters, as in Section 8 of this report. A second-order filter was thus found in which, by the addition of a last layer of thickness slightly less than half-wavelength, a characteristic is obtained as shown in Fig. 16; this characteristic is very smooth over the transmitting region.

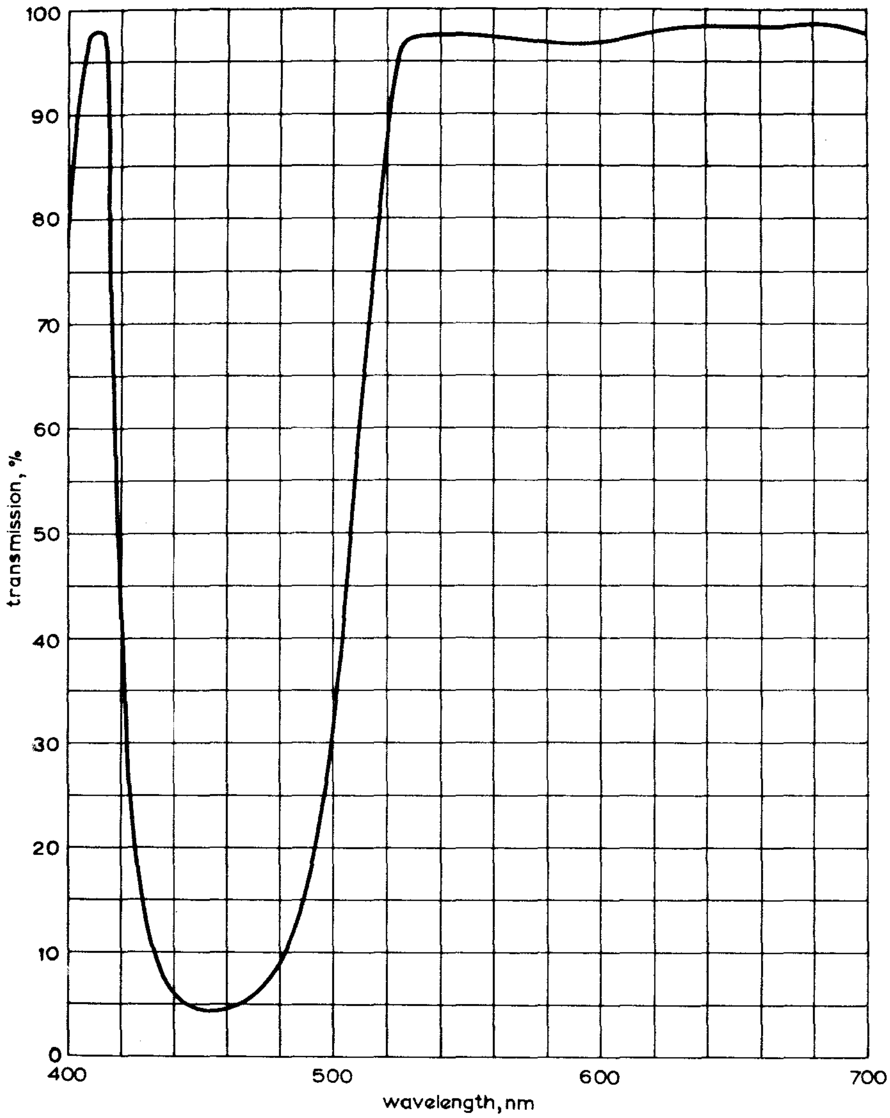

Fig. 16 - Spectral transmission characteristic of an eight layer second-order-interference filter having the last layer equal to $0{\cdot}4\,\lambda_0$

With dispersion
n_H = ZnS n_L = MgF_2 λ_0 = 460 nm

10. CONCLUSIONS

The spectral transmission characteristics of multilayer dielectric filters have been computed for various indices of the high-refractive-index layers; the effect of angle of incidence and of increasing the order of interference of the high-refractive-index layers has been evaluated; methods of flattening the response of the filters in the transmitting region are discussed. It has been shown briefly how special filters may be synthesized using a computer optimization technique.

11. REFERENCES

1. The construction of multilayer dielectric filters. BBC Research Department Report No. PH-17, Serial No. 1968/23.

2. The construction and design of dichroic mirrors. BBC Research Department Report No. T-066, Serial No. 1957/13.

JMP